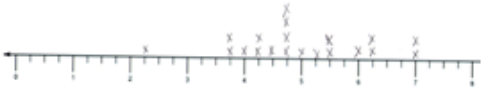
Grade 5 Module 4 – Multiplication and Division of Fractions and Decimal Fractions

Topic A: Line Plots of Fraction Measurements

- Students construct line plots by measuring the same objects using three different rulers accurate to 1/2, 1/4, and 1/8 of an inch. Students compare the line plots and explain how changing the accuracy of the unit of measure affects the distribution of points (see line plots below). This is foundational to the understanding that measurement is inherently imprecise as it is limited by the accuracy of the tool at hand.

- Students use their knowledge of fraction operations to explore questions that arise from the plotted data "What is the total length of the five longest pencils in our class? Can the half inch line plot be reconstructed using only data from the quarter inch plot? Why or why not?" The interpretation of a fraction as division is inherent in this exploration. To measure to the quarter inch, one inch must be divided into 4 equal parts, or 1 4. This reminder of the meaning of a fraction as a point on a number line coupled with the embedded, informal exploration of fractions as division provides a bridge to Topic B's more formal treatment of fractions as division.

Pencils measured to $\frac{1}{2}$ inch

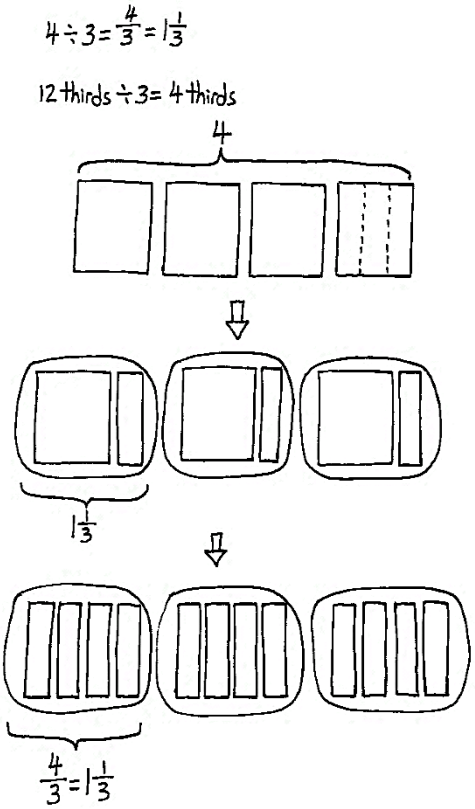


Pencils measured to $\frac{1}{4}$ inch

Topic B: Fractions as Division

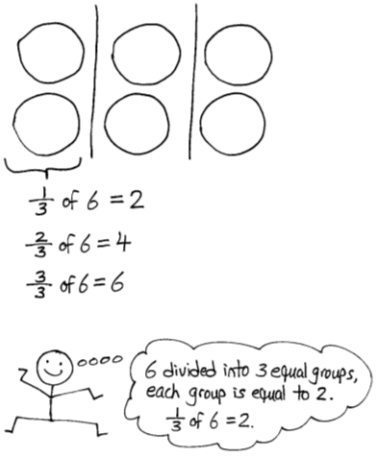
- Interpreting fractions as division is the focus of Topic B. Equal sharing with area models (both concrete and pictorial) gives students an opportunity to make sense of the division of whole numbers with answers in the form of fractions or mixed numbers (e.g., seven brownies shared by three girls, three pizzas shared by four people). Discussion also includes an interpretation of remainders as a fraction. Tape diagrams provide a linear model of these problems. Moreover, students see that by renaming larger units in terms of smaller units, division resulting in a fraction is just like whole number division.

- Topic B continues as students solve real world problems and generate story contexts for visual models. The topic concludes with students making connections between models and equations while reasoning about their results (e.g., between what two whole numbers does the answer lie?).



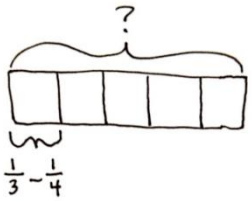
Topic C: Multiplication of a Whole Number by a Fraction

- In Topic C, students interpret finding a fraction of a set (3/4 *of* 24) as multiplication of a whole number by a fraction (3/4 × 24) and use tape diagrams to support their understandings. This in turn leads students to see division by a whole number as equivalent to multiplication by its reciprocal. That is, division by 2, for example, is the same as multiplication by 1/2.
- Students also use the commutative property to relate fraction of a set to the Grade 4 repeated addition interpretation of multiplication by a fraction. This opens the door for students to reason about various strategies for multiplying fractions and whole numbers. Students apply their knowledge of fraction of a set and previous conversion experiences (with scaffolding from a conversion chart, if necessary) to find a fraction of a measurement, thus converting a larger unit to an equivalent smaller unit (e.g., 1/3 min = 20 seconds and 2 1/4 feet = 27 inches).



Topic D: Fraction Expressions and Word Problems

- Interpreting numerical expressions opens Topic D as students learn to evaluate expressions with parentheses, such as 3 × (2/3 – 1/5) or 2/3 × (7 +9). They then learn to interpret numerical expressions such as *3 times the difference between 2/3 and 1/5* or *two thirds the sum of 7 and 9*.



"5 times the difference of $\frac{1}{3}$ and $\frac{1}{4}$"

$5 \times \left(\frac{1}{3} - \frac{1}{4}\right)$

$= 5 \times \left(\frac{4}{12} - \frac{3}{12}\right)$

$= 5 \times \frac{1}{12}$

$= \frac{5 \times 1}{12}$

$= \frac{5}{12}$

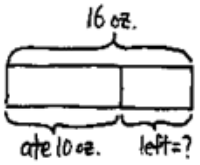
- Students generate word problems that lead to the same calculation, such as, "Kelly combined 7 ounces of carrot juice and 5 ounces of orange juice in a glass. Jack drank 2/3 of the mixture. How much did Jack drink?" Solving word problems allows students to apply new knowledge of fraction multiplication in context, and tape diagrams are used to model multi-step problems requiring the use of addition, subtraction, and multiplication of fractions.

Kim and Courtney share a 16-ounce box of cereal. By the end of the week, Kim has eaten $\frac{3}{8}$ of the box, and Courtney has eaten $\frac{1}{4}$ of the box of cereal. What fraction of the box is left?

Method 1: Kim: $\frac{3}{8}$ of $16 = \frac{3 \times \overset{2}{\cancel{16}}}{\underset{1}{\cancel{8}}} = 6$

Courtney: $\frac{1}{4}$ of $16 = \frac{1 \times \overset{4}{\cancel{16}}}{\underset{1}{\cancel{4}}} = 4$

$6 + 4 = 10$ oz.



$16 - 10 = 6$ oz.

$\frac{6}{16} = \frac{3}{8}$

$\frac{3}{8}$ of the box is left.

Method 2: $\frac{3}{8} + \frac{1}{4}$

$= \frac{3}{8} + \frac{2}{8}$

$= \frac{5}{8}$

$\frac{5}{8}$ of $16 = \frac{5 \times \overset{2}{\cancel{16}}}{\underset{1}{\cancel{8}}} = 10$ oz.

$1 - \frac{5}{8} = \frac{8}{8} - \frac{5}{8} = \frac{3}{8}$

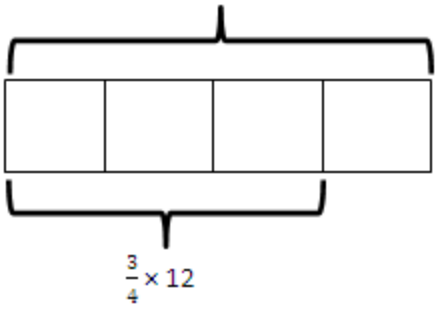
$\frac{3}{8}$ of the box is left.

Topic E: Multiplication of a Fraction by a Fraction

- Topic E introduces students to multiplication of fractions by fractions—both in fraction and decimal form. The topic starts with multiplying a unit fraction by a unit fraction, and progresses to multiplying two non-unit fractions. Students use area models, rectangular arrays, and tape diagrams to model the multiplication. These familiar models help students draw parallels between whole number and fraction multiplication and solve word problems. This intensive work with fractions positions students to extend their previous work with decimal-by-whole number multiplication to decimal-by-decimal multiplication. Just as students used unit form to multiply fractional units by wholes in Module 2 (e.g., 3.5 × 2 = 35 tenths × 2 ones = 70 tenths), they will connect fraction-by-fraction multiplication to multiply fractional units-by-fractional units. (3.5 × 0.2 = 35 tenths × 2 tenths = 70 hundredths).

$\frac{3}{4}$ of a foot = $\frac{3}{4} \times 12$ inches

1 foot = 12 inches



Express $5\frac{3}{4}$ ft as inches.

$5\frac{3}{4}$ ft = (5 × 12) inches + ($\frac{3}{4} \times 12$) inches

= 60 + 9 inches

= 69 inches

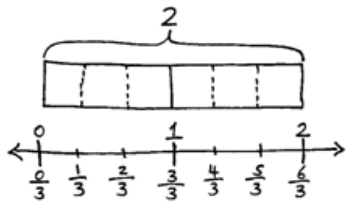
- Reasoning about decimal placement is an integral part of these lessons. Finding fractional parts of customary measurements and measurement conversion concludes Topic E. Students convert smaller units to fractions of a larger unit (e.g., 6 inches = 1/2 ft). The inclusion of customary units provides a meaningful context for many common fractions (1/2pint = 1 cup, 1/3 yard = 1 foot, 1/4 gallon = 1 quart, etc.). This topic, together with the fraction concepts and skills learned in Module 3, opens the door to a wide variety of application word problems.

Topic F: Multiplication with Fractions and Decimals as Scaling and Word Problems

- Students interpret multiplication in Grade 3 as equal groups, and in Grade 4 students begin to understand multiplication as comparison. Here, in Topic F, students once again extend their understanding of multiplication to include scaling**.** Students compare the product to the size of one factor, given the size of the other factor without calculation (e.g., 486 × 1327.45 is twice as large as 243 × 1327.45, because 486 = 2 × 243). This reasoning, along with the other work of this module, sets the stage for students to reason about the size of products when quantities are multiplied by 1, by numbers larger than 1, and smaller than 1. Students relate their previous work with equivalent fractions to interpreting multiplication by *n/n* as multiplication by 1.
- Students build on their new understanding of fraction equivalence as multiplication by *n/n* to convert fractions to decimals and decimals to fractions. For example, 3/25 is easily renamed in hundredths as 12/100 using multiplication of 4/4. The word form of *twelve hundredths* will then be used to notate this quantity as a decimal. Conversions between fractional forms will be limited to fractions whose denominators are factors of 10, 100, or 1,000. Students will apply the concepts of the topic to real world, multi-step problems

Topic G: Division of Fractions and Decimal Fractions

- Topic G begins the work of division with fractions, both fractions and decimal fractions. Students use tape diagrams and number lines to reason about the division of a whole number by a unit fraction and a unit fraction by a whole number. Using the same thinking developed in Module 2 to divide whole numbers, students reason about how many *fourths* are in 5 when considering such cases as 5 ÷ 1/4. They also reason about the size of the unit when 1/4 is partitioned into 5 equal parts: 1/4 ÷ 5. Using this thinking as a backdrop, students are introduced to decimal fraction divisors and use equivalent fraction and place value thinking to reason about the size of quotients, calculate quotients, and sensibly place the decimal in quotients.



$2 \div \frac{1}{3} = 6$

She can make 6 bags.

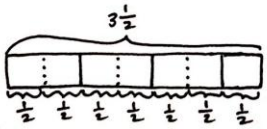
Topic H: Interpretation of Numerical Expressions

- The module concludes with Topic H, in which numerical expressions involving fraction-by-fraction multiplication are interpreted and evaluated. Students create and solve word problems involving both multiplication and division of fractions and decimal fractions.

Lucia has 3.5 hours left in her workday as a car mechanic. Lucia needs – of an hour to complete one oil change.

a. How many oil changes can Lucia complete during the rest of her workday?
b. Lucia can complete two car inspections in the same amount of time it takes her to complete one oil change. How long does it take her to complete one car inspection?
c. How many inspections can she complete in the rest of her workday?

2a) How many half hours in 3.5 hours?

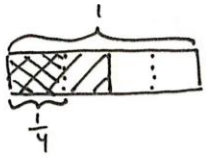


$3.5 \div \frac{1}{2}$

$= 3\frac{1}{2} \div \frac{1}{2}$

There are 2 halves in 1 w)
There are 6 halves in 3 w)
There is 1 half in 1 half
There are 7 halves in $3\frac{1}{2}$.

2b) $\frac{1}{2} \div 2 = \frac{1}{4}$



Lucia can complete one car inspection in $\frac{1}{4}$ hour.

2c) $7 \times 2 = 14$

Lucia can complete twice as many inspections as oil changes, so she can complete 14 inspections in 3.5 hours.

2c) How many $\frac{1}{4}$ hours are in 3.5 hours?

$3.5 \div \frac{1}{4}$

$= 3\frac{2}{4} \div \frac{1}{4}$

There are 12 fourths in 3.
There are 2 fourths in $\frac{2}{4}$.
There are 14 fourths in $3\frac{2}{4}$.

Lucia can complete 14 inspections in 3.5 hours.